# CELL MODEL OF THE KINETICS OF GRANULE FORMATION IN 

## FLUIDIZED-BED EQUIPMENT

G. A. Minaev, B. S. Dmitrievskii, and D. A. Oskalenko

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The results of numerical and experimental investigation of the kinetics of granulation in fluidized beds are outlined.

In constructing a model of the kinetics of granule formation in a fluidized bed, the spraying region is not separated in most works, but is taken to be equal to the volume of the whole bed. The bed is then described as an idealmixing cell, and the continuity equation for the density of the size distribution of the bed particles is used to calculate the kinetics of granule formation [1]. The assumption that the liquid product is sprayed simultaneously onto all the bed particles is very rough, since the volume of the spraying zone in all types of atomizers is less than the bed volume, amounting to $5-20 \%$.

In describing the size distribution of the finished product within the framework of the single-cell model, the concept of the fractional growth rate $\lambda(r)$ is introduced, and the form of its dependence on the radius is determined. The fractional growth rate is an integral quantity, and therefore its dependence on the particle radius obtained experimentally yields little information and does not reflect the experimental conditions.

Attempts were made in $[2,3]$ to isolate the spraying zone and take account of its features in calculating the kinetics of granule formation. The spraying zone was created by operation of a pneumatic force pump at the side wall of the apparatus. The motion of solid phase in this zone was described by an ideal-mixing cell. The same cell also described the remaining zone of the bed, in which there was no spraying. The two zones were connected by a direct flux of solid phase from the bed zone to the spraying zone and an inverse flux consisting of granules increasing in size in the spraying zone. To estimate the mass of the fluxes and the volume (mass) of the granules in the spraying zone, parameters such as the mean time between successive arrivals of the particles in the spraying zone and the mean residence time of the granules in this zone were used in the model. To describe granule growth, the parameter $\lambda(t)$ was introduced - the growth rate in the spraying zone. Solution of the system of differential and integral equations obtained in [2] is only possible by numerical methods.

The aim of the present work is to develop and solve a mathematical model of the granulation process, taking account of the spraying-zone parameters, and to test its adequacy against the actual process of granule growth.

To write the equations of the model, it is necessary to know the spraying-zone parameters. These parameters are: the granule mass of granules in the spraying zone at each instant of time; the mass flux of granules through the spraying zone; the distribution of the granules with respect to the residence time; and the growth rate of granules in this zone.

In fluidized-bed equipment, different types of atomizers (force pumps) are used to supply the solution at the surface and inside the bed. For the case when spraying of the liquid product is undertaken in the volume of the bed, all types of atomizer create jet flows of directed motion of the solid-phase particles.

Each set of jet-flow conditions is characterized by a definite - depending on the volume and geometry circulational loop, formed by mobile particles between the attenuated central region and the peripheral region (the bed). The jet flows in different conditions differ also in mass of the particles passing through the circulational loop and in the residence time of the particles there. The volume (mass) of the particles subjected to the drops of the sprayed product is equal to the volume of the circulational loop, to an accuracy of 4-11.5\% [4]. The parameters of the spraying zone (geometry, particle mass, mass flow rate of particles through the zone, etc.) may be determined for each of the jetflow conditions from the gas-phase momentum according to the results of $[5,6]$.

For the spraying zone in bubble flow conditions of the solid phase, the penetration of particles from the volume of the bed and their entrainment from the spraying zone is negligible. Circulational flow of particles similar to the idealmixing model is formed in the zone. For developed bubble conditions, the solid-phase flux through the spraying zone

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Fig. 1. Two-zone model of fluidized bed of granulator.


Fig. 2. Size distribution of particles in granulation with no raw-material supply and steady conditions of solid-phase flow in the spraying zone with $\mathrm{Q}_{\mathrm{B}}=5.5 \mathrm{~kg} / \mathrm{sec}$ : 1) $\mathrm{N}_{\mathrm{c}}=10$; 2) 6 ; 3) 3 ; 4) 1 ; 5) single-zone model of fluidized bed; 6) experiment; $V(r, t) ; d, m$.
increases in comparison with bubble conditions. Steady conditions and local-gushing conditions are characterized by a large mass flow rate of solid phase in comparison with the two sets of conditions noted above; in terms of the residence time of the particles they resemble the ideal-displacement model. Then it is natural to use a cell model to des ribe all the various conditions. The spraying zone and the adjacent zone of the bed may be shown schematically in the form of a system of cells connected by a mass flux of particles (Fig. 1).

Particles of raw material enter the bed zone and the finished product is discharged. In each cell of the spraying zone, liquid product is applied; the particles of each cell are characterized by a particular size distribution described by a separate continuity equation, and the whole of the reaction zone of the apparatus is described correspondingly by a system of equations forming the mathematical model of the kinetics of the granulation process.

In constructing the mathematical model, the following assumptions are made: 1) the particles are sphe ical; 2) the liquid applied to a particle flows over it in a thin film; 3) the volume of the spraying zone and the mass flow rate of particles through it are constant over time; 4) the distribution of particles with respect to the residence time in the zone is approximated by the cell model, and the bed zone by an ideal-mixing cell; 5) the growth rate $\lambda(t)$ in the spraying zone for each cell is proportional to the particle surface in this cell.

The particle mass in a single cell of the spraying zone and the quantity of dry material applied to the particle of the cell may be defined as

$$
\begin{equation*}
G_{\mathbf{s} i}=G_{\mathbf{s}} / N_{\mathrm{c}}, \quad Q_{\mathrm{d} i}=Q_{\mathrm{d}} / N_{\mathrm{c}} . \tag{1}
\end{equation*}
$$

Assuming that the initial distribution of the particles entering the first cell of the spraying zone is identical to the distribution of particles in the bed $V(r, t)$, the number of such particles is found from the expression

$$
\begin{equation*}
n_{\mathrm{inl}}=\frac{Q_{\mathbf{S}}}{\frac{4}{3} \pi \rho \int_{0}^{\infty} r^{3} V(r, t) d r} \tag{2}
\end{equation*}
$$

The growth rate, the change in particle size on account of spraying, the number of particles at the exit, and the change in the number of particles for the i-th cell of the spraying zone are determined from the following system of equations

$$
\begin{align*}
& G_{\mathbf{S} i}+Q_{\mathrm{d}_{i}}=N_{\mathrm{S}_{i}} \frac{4}{3} \pi \rho \int_{0}^{\infty}\left(r+\lambda(t)_{i}\right)^{3} U_{i}(r, t) d r,  \tag{3}\\
& \frac{\partial U_{i}(r, t)}{\partial t}+\lambda(t)_{i} \frac{\partial U_{i}(r, t)}{\partial r}=\frac{n_{\text {in }}}{N_{\mathbf{s}^{i}}}\left(U_{i-1}(r, t)-U_{i}(r, t)\right),  \tag{4}\\
& n_{\text {out } i}=\frac{Q_{\mathbf{s}}+Q_{\mathrm{d}} i}{\frac{4}{3} \pi \rho \int_{0}^{\infty} U_{i}(r, t) d r},  \tag{5}\\
& n_{\text {ini } i+1}=n_{\text {out } i}, \quad U_{0}(r, t)=V(r, t),  \tag{6}\\
& \frac{d N_{\mathbf{s}_{i}}}{d t}=n_{\mathrm{in}_{i}}-n_{\text {out } i} \text { for } i=1 \text { to } N \mathrm{c} \text {. } \tag{7}
\end{align*}
$$

The initial and boundary conditions for Eqs. (3)-(7) are as follows

$$
\begin{gathered}
U_{i}^{0}=V^{0}(r, t), N_{\mathrm{S} i}^{0}=\frac{G_{\mathrm{S} i}}{\frac{4}{3} \pi \rho \int_{0}^{\infty} r^{3} V^{0}(r, t) d r} \\
U_{i}(0, t)=0 \text { for } i=1 \text { to } N_{\mathrm{c}}
\end{gathered}
$$

Solving the system of equations obtained systematically for each cell, the parameters of the inverse flux of particles leaving the spraying zone are determined. The number of particles in the output flux (Fig. 1 ) is $n_{o u t N_{c}}$, and the size distribution of the particles coincides with $\mathrm{U}_{\mathrm{N}_{\mathrm{c}}}(\mathrm{r}, \mathrm{t})$.

To close the system of equations of the model, the fluxes of raw material and discharged product are defined [2] as

$$
\begin{align*}
& Q_{r}=n_{\mathbf{r}} \frac{4}{3} \pi \rho \int_{0}^{\infty} r^{3} V_{\mathbf{r}}(r, t) d r  \tag{8}\\
& Q_{\mathrm{dis}}=n_{\mathrm{dis}} \frac{4}{3} \pi \rho \int_{0}^{\infty} r^{3} V(r, t) d r \tag{9}
\end{align*}
$$

From the condition that the mass of the bed is constant, the mass of discharged product must satisfy the following equation

$$
\begin{equation*}
Q_{\mathrm{dis}}=Q_{\mathrm{s}}+\sum_{i=1}^{N_{\mathrm{c}}} Q_{\mathrm{di}} \tag{10}
\end{equation*}
$$

The change in the number of particles and their size distribution for the bed volume is determined by the equations

$$
\begin{gather*}
\frac{d N}{d t}=n_{\mathbf{r}}-n_{\mathrm{dis}}  \tag{11}\\
\frac{\partial V(r, t)}{\partial t}=\frac{n_{\mathrm{out} N_{\mathrm{c}}}}{N-\sum_{i=1}^{N_{\mathrm{C}}} N_{\mathbf{S}}}\left(U_{N_{\mathrm{C}}}(r, t)-V(r, t)+\right. \\
+\frac{n_{\mathrm{r}}}{N-\sum_{i=1}^{N_{\mathrm{c}}} N_{\mathbf{S} i}}\left(V_{\mathrm{r}}(r, t)-V(r, t)\right) \tag{12}
\end{gather*}
$$

with the following initial and boundary conditions

$$
\begin{gathered}
V^{0}(r, t)=V(r, 0), \quad N^{0}=\frac{G_{\mathrm{be}}}{\pi \frac{4}{3} \rho \int_{0}^{\infty} r^{3} V^{0}(r, t) d r} \\
V(0, t)=0
\end{gathered}
$$



Fig. 3. Dependence of the number of cells in the model describing the spraying zone on: a) the jet-flow conditions: I) bubble conditions of solidphase flow; II) developed bubble flow; III) local gushing; IV) steady flow; 1) $Q_{g}, \mathrm{~kg} / \mathrm{sec} ; 2$ ) $G_{g}, \mathrm{~kg}$; b) on the mean particle diameter of the initial bed packing $\mathrm{d}_{\mathrm{me}}{ }^{0}, \mathrm{~m}$, with $\mathrm{G}_{\mathrm{be}}{ }^{0}=5.5 \mathrm{~kg}(1)$ and on the mass of the initial bed packing $G_{b e}{ }^{0}, \mathrm{~kg}$, with $\mathrm{d}_{\mathrm{me}}{ }^{0}=2.8 \cdot 10^{-3} \mathrm{~m}$ (2) with a constant flow rate of the atomizing air of $0.013 \mathrm{~m}^{3} / \mathrm{sec}$ for cases 1 and 2 .


Fig. 4. Granule growth in granulation with raw-material supply and steady flow conditions of the solid phase in the spraying zone: a) distribution of the granules with respect to the size of the initial packing 1 (1) and with respect to the size of the initial packing 2 (2); 3-6) variation in distribution density of the granules in the course of growth for initial packing 1 ; b) variation in mean diameter $d_{m e} m$, of bed granules with initial packing 1 (1) and 2 (2) with identical raw-material size; $V(r, t) ; d$, $\mathrm{m} ; \mathrm{t}$, sec.

Differential Eqs. (4), (7), (11), and (12) are solved by the grid method, replacing the derivatives by difference analogs. The time interval in which the calculation is performed is determined from the condition of stabilizaticn of the granular composition of the bed.

To test the adequacy of the proposed model, experiments are undertaken on the granulation of a yeast suspension in a pilot plant. The apparatus takes the form of a cylindroconical unit with a diameter of the gas-distributor lattice of 250 mm and a height of 1500 mm . At the side wall of the apparatus, there is a two-channel pneumatic internal-mixing force pump with an interchangeable nozzle and an adjustable gap between the liquid and air channels. Granuation is conducted without supply of raw material and with continuous discharge of the finished product, in such a fashion that the weight of the bed remains constant in the course of spraying. Product samples are taken at the level of the gasdistributor lattice. Fluidization of the bed is by means of hot air (at a temperature of $170^{\circ} \mathrm{C}$ ). Changing $Q_{s}$ by means of the air flow rate into the force pump ensures the reproduction of all four jet-flow conditions.

Experimental and theoretical histograms of the granular composition of the bed are compared in Fig. 2 for steady flow conditions of the solid phase at time $t=20 \mathrm{~min}$. The theoretical curves are obtained by approximating the spraying zone with different numbers of cells. As is evident from Fig. 2, theoretical curves 1 and 2 with $N_{c}=10$ and 6 are closest to the experimental data (with an accuracy of $3-15 \%$ ). Theoretical curve 5 , which corresponds to the condition that the spraying zone coincides in volume with the bed zone and is described by a single ideal-mixing cell exhibits serious discrepancy with the experimental data: $40 \%$ and more. Hence, representing the bed zone and the spraying zone by a single ideal-mixing cell leads to large errors.

A relation between the jet-flow parameters and the number of spraying-zone cells $\mathrm{N}_{\mathrm{c}}$ is observed experimentally. It is evident from Fig. 3 that increase in $G_{s}, Q_{B}$ is associated with transition from bubble jet-flow conditions in the spraying zone to steady conditions, and the model parameter $\mathrm{N}_{c}$ changes from a few units in bubble conditions to a few tens in steady conditions.

Increase in bed mass influences $N_{c}$ as much as this affects the jet-flow conditions of the solid phase in the spraying zone. Turning to the dependence of $\mathrm{N}_{\mathrm{c}}$ on the size distribution of the particles in the initial bed packing, experiment shows that the mean diameter of the initial bed filling and the form of the function $V^{0}(r, t)$ have practically no influence on $N_{c}$ (Fig. 3b), although there are different size distribution functions of the granules here at the moment that the granulator reaches steady operating conditions. This confirms the stability of the jet-flow conditions with change in mean diameter of the bed particles over a wide size range noted in [6].

The agreement of the theoretical and experimental data on the kinetics of granulation with no raw-material supply allows this approach to be extended to the numerical investigation of granulation in the presence of raw-material supply. The results of calculating the kinetics of granulation in the case of raw-material supply for a pilot plant are shown in Fig. 4. The time interval between the curves is $t=10 \mathrm{~min}$ with continuous supply of the raw-material and continuous unloading of the finished product. The value of $\mathrm{N}_{\mathrm{c}}$ is determined from Fig. 3 for steady conditions of solidphase flow.

As is evident from Fig. 4, the initial single-mode curve 1 is transformed, as a result of granule growth and rawmaterial supply, first to a bimodal curve and then back to a single-mode curve (curve 6) when the apparatus reaches steady conditions. At this time, the particles of the initial loading are almost completely discharged. The kinetics of variation in granular composition of the bed in the transitional period depends on the initial distribution $V^{0}(r, t)$ of the particles in the bed. The $60 \%$ difference in mean diameter of the initial granules (characterized by curves 1 and 2 ) leads at time $t=40 \mathrm{~min}$ to a difference in the mean diameters by $100 \%$ at time $t=40 \mathrm{~min}$, and by $150 \%$ at the time of stabilization of the bed granular composition. The final granule distribution for the initial dimensions of the initial packings 1 and 2 when the granulator reaches steady conditions may correspond to the requirements imposed on the particle size, but the intermediate state of the bed may lead to defluidization and emergency shutdown of the unit. Therefore, analysis of the stability of the granulation process with multiple numerical investigation of the model is required.

Numerical solution of the proposed mathematical model permits the determination of the final size distribution of the granules and the granular composition of the bed in the transitional (nonsteady) conditions and analysis of its stability. Repeated calculation of the model equations also permits the solution of optimization problems: determination of the characteristics of the fluxes of raw material and discharged product and the characteristics of the spraying zone in solving the problem of most rapid stabilization of the granulator and optimization of the parameters of the auxiliary equipment. The universality of the jet flows allows the given model to be recommended for the calculation of the granulation kinetics for various types of atomizers employed in fluidized-bed granulators.

## NOTATION

$G_{b e}$, bed mass; $N_{c}$, number of cells in spraying zone; $G_{s}$, particle mass in spraying zone; $Q_{B}$, mass flow rate of particles through spraying zone; $Q_{d}$, quantity of dry materials applied to fluidized-bed granules; $n_{r}, n_{\text {outl }}, n_{\text {dis }}, N, N_{s i}$, number of raw-material particles, number of particles at the exit from the spraying zone, number of particles of discharged product, number of particles in the bed, and number of particles in the $i-t h$ cell of the spraying zone; $V(r$, $t), V_{r}(r, t), U_{i}(r, t)$, distribution density of particle radii in bed zone, raw-material flux, and $i$-th cell of spraying zone; $\rho$, density of dry granulated product; $\lambda(\mathrm{t})_{\mathrm{i}}$, growth rate of granules in i -th cell of spraying zone.

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